Work and Energy

Newton's laws are sufficient to describe the motion of all mechanical systems. However, scientists have formulated other descriptions based on Newton's laws that have proved more useful. For example, acceleration, in terms of which Newton's second law is formulated, is not as basic, is less intuitive, and is more mathematically difficult to deal with than velocity. Thus, we will next develop the idea of energy, which is formulated in terms of velocity rather than acceleration. We will say that by exerting a force on the object through a distance, work is done, and that work goes into changing kinetic energy (the word *energy* means capacity for doing work). The word *Kinetic* comes from the Greek word *kinētikos* (to move), so *kinetic energy* is energy of motion.

Work. To understand energy we must define work. If we are going to formulate our treatment in terms of velocity, we will involve either distance moved or time. Also, we must talk about force. A force may just change the *direction* of motion, as in the case of circular motion, where the force is perpendicular to the motion. Thus we must take into account only the component of the force in the direction of the motion and disregard the part perpendicular to the motion. Also, because the longer the force acts, the greater the effect, we must take into account the distance moved. So work will be defined in terms of force and distance moved.

Work $\equiv$ the component of the force in the direction of the motion $\times$ the distance x moved $= F_{//} \times x$ **Note:** The component of the force perpendicular to the motion $F_{\perp}$ does no work.

Question: Can work ever be negative? Answer: Yes, if the force doing the work is opposite to the motion.

Example: Let's say that I do the following with my briefcase, which weighs 12 N. First, (a) I lift it 1 m at constant speed, then (b) I hold it for 20 s, then (c) I carry it horizontally 2 m at constant speed, then, (d) I lower it back to the floor at constant speed. How much work did I do in each case? Answers: (a) W = (12)(1) = 12 J, (b) zero (no motion), (c) zero (the force is perpendicular to the motion), (d) (–12)(1) = –12 J.

Kinetic Energy. The kinetic energy (KE) of a mass m, moving with speed v is defined as KE $\equiv \frac{1}{2}mv^2$.

Let us derive the relationship between work and kinetic energy for a mass m moving along a straight line at constant acceleration a, under the action of a single force F:

Applying the kinematical equation $v_f^2 = v_0^2 + 2ax$ and $\Sigma F = ma$, *we get:*

$$\text{Work} = F\,x = m(ax) = m(v_f^2 - v_0^2)/2$$
$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$$
$$= \text{final KE} - \text{initial KE}$$
$$= \text{change in KE} = \Delta\text{KE}$$

It can be shown that, in general, the work done by all forces equals the change in KE:

$$\text{Work}_{\text{total}} = \Delta\text{KE}$$

Example: A mass m rests on a horizontal frictionless surface. Let's say that I exert a horizontal force P on the mass. By Newton's second law $\Sigma F = ma$, the mass will accelerate, or equivalently, it will speed up.

If the distance the mass is moved is d, then the work will be $P \times d$. Let's say that P = 36 N, d = 1 m, and m = 2 kg. From this information, we can calculate the speed of the object after being moved through a distance d = 1 m:

$$\text{Work} = P_{//} \times d = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2.$$

$$36 \times 1 = \frac{1}{2}(2)v_f^2 - \frac{1}{2}(2)(0)^2$$

$$v_f^2 = 36$$

$$v_f = 6 \text{ m/s}.$$

Example: A mass m is dropped from a height h = 5 m. The work done by gravity will be $mg \times h$. From this information, we can calculate the speed of the object after it falls through a distance h = 5 m:

$$\text{Work} = F_{//} \times x = \frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_0^{\,2}$$

$$\cancel{m}g \times h = \frac{1}{2}\cancel{m}v_f^{\,2} - \frac{1}{2}\cancel{m}(0)^2$$

$$(10)(5) = \frac{1}{2}v_f^{\,2}$$

$$v_f^{\,2} = 100$$

$$\Rightarrow v_f = 10 \text{ m/s.}$$

We take the positive root because we are only interested in the speed (we know the direction).

Note: The fact that the m's canceled shows that all masses will have the same speed when dropped through the same height.

Example: Compare the following:

(1) Push a mass a distance h at constant speed in a straight line on the table under friction. Where did the energy go?

Answer: It went into thermal energy, which is very hard to retrieve.

(2) Lift a mass m through a height h. The work I do will be $mg \times h$. Where did the energy go?

Answer: It is potentially available because, if I drop the mass, gravity will do $mg \times h$ of work on the mass—the same amount of work that it took to lift it. Therefore, by my lifting the mass, the energy I expended was not lost but went into potential energy = mgh.

The word *potential* comes from the word *potent* (having power), so *potential* means having power that has not yet been brought into being.

Conservation of Energy. We also say that during its way down, the mass lost potential energy, which was converted into kinetic energy. Thus, in this case (gravity is the only force doing work), mechanical energy is conserved (not lost).

Note that the work done by gravity (and consequently, the change in PE) does not depend on the path but only on the vertical distance a mass moves. When a mass m rises by h, its PE increases by mgh, and when it lowers by h, its PE decreases by mgh. Thus $\Delta\text{PE} = \pm mgh$.

In general, conservation of mechanical energy can be stated:

If gravity is the only fore doing work, then ΔKE + ΔPE = 0.

Consider again the example of a mass m that is dropped from a height h = 5 m:

Energy is conserved because gravity is the only force doing work, so

$$\Delta\text{KE} + \Delta\text{PE} = 0$$

$$\frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_0^{\,2} - mgh = 0$$

The m's again cancel, and $v_0 = 0$.

$$\text{Thus, } \frac{1}{2}v_f^{\,2} - (10)(5) = 0$$

$$v_f^{\,2} = 100$$

$$\Rightarrow v_f = 10 \text{ m/s, as before.}$$